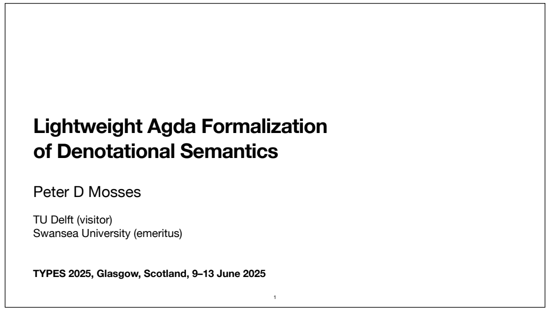
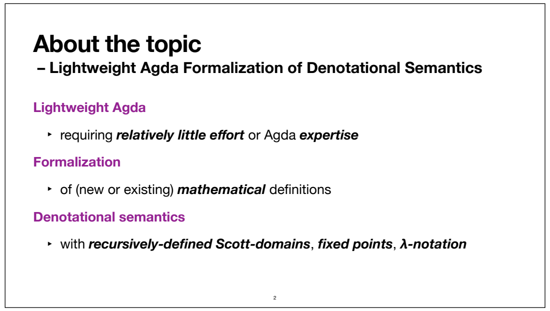


1

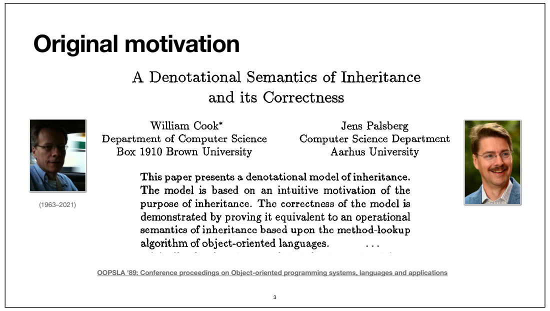
Thank you for still being here!

This is a lightweight talk about work in progress



2

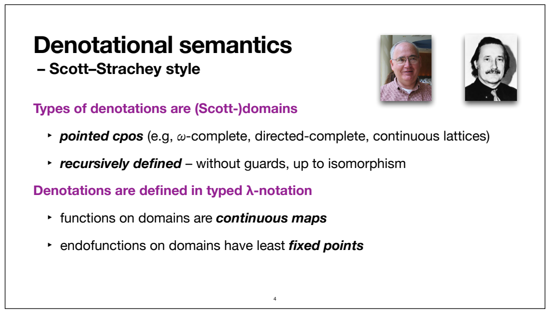
Let me start by clarifying the topic of the talk



3

This denotational semantics was published in 1989, and hadn't been mechanically checked.

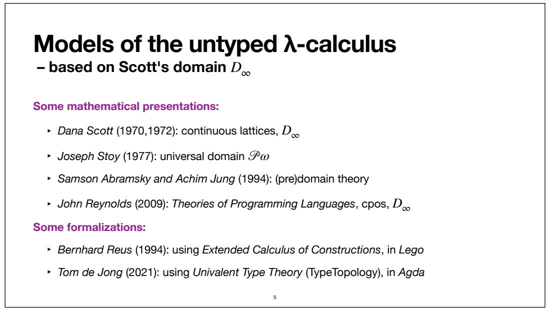
I expected it to be quite straightforward to formulate the definitions and proofs in Agda...



4

Denotational semantics isn't as popular these days as it was in the 70s and 80s

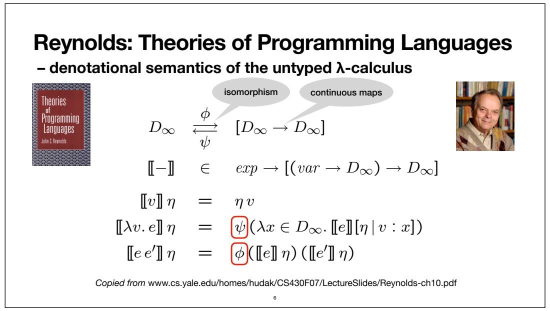
Let me briefly recall the main features



5

Dana Scott initially tried to prove that the untyped lambda-calculus has no set-theoretic models
but then he discovered the bilimit construction of the D-infinity model…

Apart from mathematical presentations of the model, some formalizations have been developed



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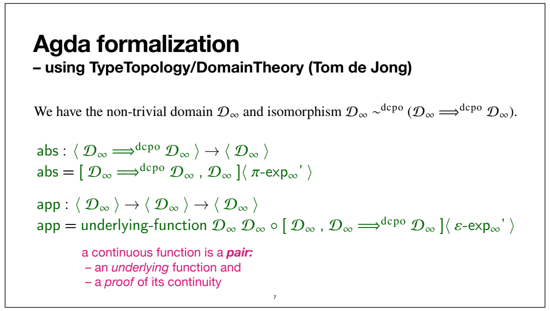
Here is a mathematical presentation of a denotational semantics of the untyped lambda-calculus from 2009

In denotational semantics of larger programming languages, isomorphisms are usually left implicit



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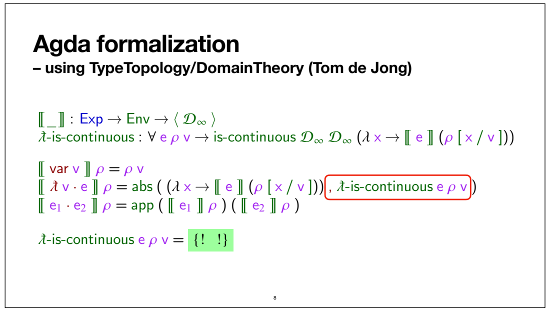
Tom de Jong's formalization is based on directed-complete posets.



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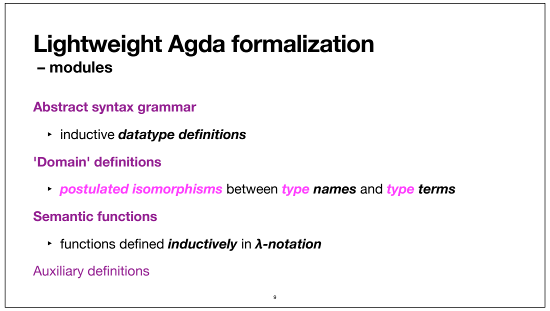
The proof of the proposition that lambda is continuous isn't very deep, but takes 3 pages in John Reynolds book

I would personally find it an excessive amount of work to formalize the proof in Agda...



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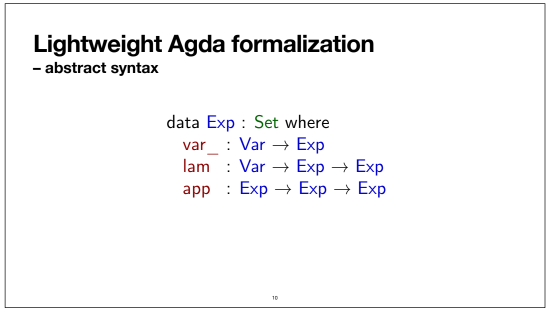
... so I've developed a lightweight approach

The next few slides illustrate the approach. The Agda definitions are included in the abstract, and available online



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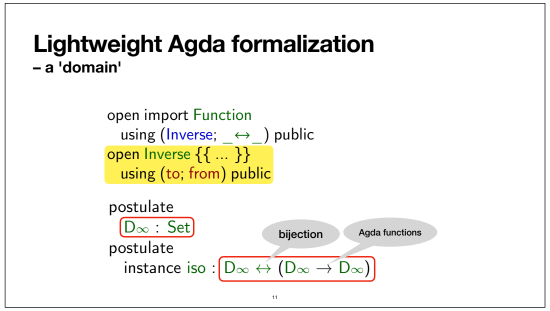
In Scott–Strachey style, abstract syntax is defined by a context-free grammar. Its formalization in Agda is a corresponding inductive datatype.

For simplicity, this datatype uses ordinary functional notation for the constructors.

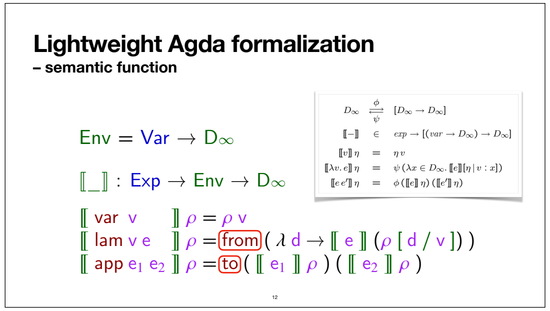


11

Here we assume D-infinity to be an ordinary Agda type with a bijection to the type of all Agda functions on D-infinity

In this example, the assumptions are satisfied when D-infinity has a single element.
In all other examples, the corresponding assumptions are unsatisfiable.

The highlighted Agda magic declares the inverse functions of the bijection, which are all we need...



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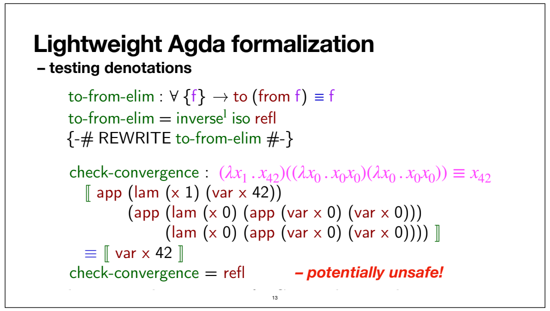
The semantic function corresponds directly to that defined by Reynolds!

The Agda type-checker insists on making the bijection explicit



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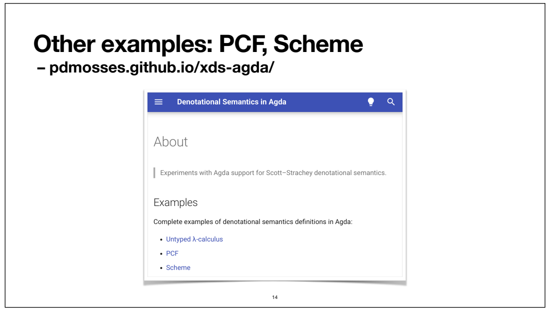
Agda can also check whether terms have equivalent denotations



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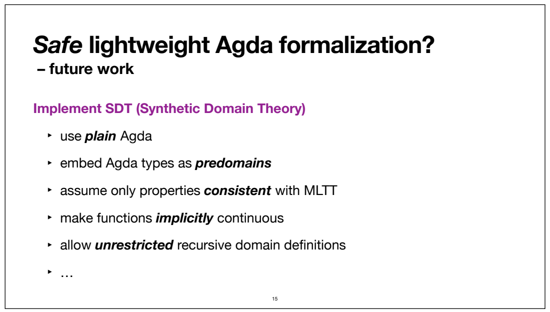

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